Towards the effective 2-topos

Jacopo Emmenegger

Università di Genova

joint work with Steve Awodey

with thanks to M. Anel, R. Barton, J. Frey, and P. Rosolini

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Introduction

Useful features of the effective topos $\mathcal{Eff}$:

- It is a(n elementary) topos, in particular locally cartesian closed.
- All functions on natural numbers are Turing-computable (Church Thesis).
- It contains a small subcategory which is complete but not a preorder (modest sets).

So $\mathcal{Eff}$ allows to construct realisability models of extensional type theories with an impredicative universe of types.

Hyland. The effective topos. 1982
Hyland. A small complete category. 1988
Hyland, Robinson, Rosolini. The discrete objects in the effective topos. 1990

Introduction

Current line of research: to identify higher versions of $\mathcal{E}\!f\!f$ that

- interpret intensional type theories with a univalent impredicative universe,
- and provide examples of elementary higher toposes.

Desiderata for every higher-dimensional effective topos:
it should contain $\mathcal{E}\!f\!f$ as the full subcategory of 0-types (=h-sets).

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Desiderata for every higher-dimensional effective topos:
it should contain $\mathcal{E}\!f\!f$ as the full subcategory of 0-types (=h-sets).

Straightforward options (taking simplicial or cubical objects in $\mathcal{E}\!f\!f$ or $\mathcal{A}sm$) add 0-types: $\mathcal{E}\!f\!f$ is already a completion (under quotients of equivalence relations), so taking $\mathcal{E}\!f\!f^{\Delta^{\mathrm{op}}}$ adds those colimits again at the level of 0-types.
An eq. relation $E \rightrightarrows X$ in $\mathcal{E}\!f\!f$ has two distinct quotients in $\mathcal{E}\!f\!f^{\Delta^{\mathrm{op}}}$:
the (discrete simplicial set on the) quotient X/E and (the nerve of) $E \rightrightarrows X$.

Today: a 2-dimensional proposal, using groupoids.

Awodey, E. Toward the effective 2-topos. arXiv:2503.24279

The effective topos as a colimit completion

- $\mathcal{E}\!f\!f$ is the exact completion of the regular category of assemblies $\mathcal{A}sm$:

$$\mathcal{E}\!f\!f = (\mathcal{A}sm)_{\mathsf{ex/reg}}$$

  Carboni, Freyd, Scedrov. A Categorical approach to Realizability and Polymorphic Types. 1988

- since $\mathcal{A}sm$ is the regular completion of the lex category of partitioned assemblies $\mathcal{PA}sm$, we also have:

$$\mathcal{E}\!f\!f = (\mathcal{A}sm)_{\mathsf{ex/reg}} = (\mathcal{PA}sm)_{\mathsf{ex/lex}}$$

  Robinson, Rosolini. Colimit completions and the effective topos. 1990
  Carboni. Some free constructions in realizability and proof theory. 1995

As such, $\mathcal{E}\!f\!f$ embeds into presheaves on $\mathcal{PA}sm$,
and can be characterised as a subcategory of $\widehat{\mathcal{PA}sm}$.

Lack. A note on the exact completion of a regular category, and its infinitary generalizations. 1999
Hu, Tholen. A note on free regular and exact completions and their infinitary generalizations. 1996

$$\begin{tikzcd}
\mathcal{PA}sm \arrow[r, "\mathsf{y}"] \arrow[d] & \widehat{\mathcal{PA}sm} \\
\mathcal{A}sm \arrow[r] & \mathcal{E}\!f\!f \arrow[u]
\end{tikzcd}$$

The effective topos as a colimit completion

$$\begin{tikzcd}
\mathcal{PAsm} \arrow[r] \arrow[dd, "\mathsf{y}"'] & \mathcal{Asm} \arrow[d] \\
& \mathcal{Eff} \arrow[dl] \\
\widehat{\mathcal{PAsm}} &
\end{tikzcd}$$

$$\mathsf{y}Q \twoheadrightarrow A \rightarrowtail \mathsf{y}P$$

$$B \rightrightarrows A \twoheadrightarrow X$$

The effective topos as a colimit completion

$$
\begin{tikzcd}
\mathcal{PAsm} \arrow[r] \arrow[dd, "\mathsf{y}"'] & \mathcal{Asm} \arrow[d] \\
& \mathcal{Eff} \arrow[d] \\
\widehat{\mathcal{PAsm}} & \mathcal{Cpt}(\widehat{\mathcal{PAsm}}) \arrow[l]
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathsf{y}Q \arrow[r, two heads] & A \arrow[r, tail] & \mathsf{y}P
\end{tikzcd}
$$

$$
\begin{tikzcd}
B \arrow[r, shift left] \arrow[r, shift right] & A \arrow[r, two heads] & X
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathsf{y}P \arrow[r, two heads] & K
\end{tikzcd}
$$

The effective topos as a colimit completion

$$
\begin{tikzcd}
\mathcal{PAsm} \arrow[r] \arrow[ddd, "\mathsf{y}"'] & \mathcal{Asm} \arrow[d] \\
& \mathcal{Coh}(\widehat{\mathcal{PAsm}}) \arrow[d] \\
& \mathcal{Eff} \arrow[d] \\
\widehat{\mathcal{PAsm}} & \mathcal{Cpt}(\widehat{\mathcal{PAsm}}) \arrow[l]
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathsf{y}Q \arrow[r, two heads] & {\color{red}A} \arrow[r, tail] & \mathsf{y}P
\end{tikzcd}
$$

$$
\begin{tikzcd}
\mathsf{y}P \arrow[r, two heads] & {\color{red}C}
\end{tikzcd}
$$

and

$$
\begin{tikzcd}
K' \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & C \arrow[d, tail] \\
K \arrow[r] & C \times C
\end{tikzcd}
$$

$$
\begin{tikzcd}
B \arrow[r, shift left] \arrow[r, shift right] & A \arrow[r, two heads] & {\color{red}X}
\end{tikzcd}
$$

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\begin{tikzcd}
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\end{tikzcd}
$$

The effective topos as a colimit completion

$$\begin{tikzcd}
\mathcal{PAsm} \arrow[r] \arrow[ddd, "\mathsf{y}"'] & \mathcal{Asm} \arrow[d] \\
 & \mathcal{Coh}(\widehat{\mathcal{PAsm}}) \arrow[d, equal] \\
 & \mathcal{Eff} \arrow[d] \\
\widehat{\mathcal{PAsm}} & \mathcal{Cpt}(\widehat{\mathcal{PAsm}}) \arrow[l]
\end{tikzcd}$$

$\mathsf{y}Q \twoheadrightarrow A \rightarrowtail \mathsf{y}P$

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$$\begin{tikzcd}
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K \arrow[r] & C \times C
\end{tikzcd}$$

$B \rightrightarrows A \twoheadrightarrow X$

$\mathsf{y}P \twoheadrightarrow K$

Outline

Give:

1. Definition of coherent groupoid
2. Quillen model structure on $\mathbf{Gpd}(\widehat{\mathcal{PAsm}})$

such that:

$$\begin{tikzcd}
\mathcal{Eff} \arrow[r, hook] \arrow[d, hook] \arrow[dr, phantom, "\lrcorner", very near start] & \mathbf{Gpd}_0(\widehat{\mathcal{PAsm}}) \arrow[d, hook] \\
\mathbf{CohGpd}(\widehat{\mathcal{PAsm}}) \arrow[r, hook] & \mathbf{Gpd}(\widehat{\mathcal{PAsm}})
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\mathbf{Eff}_2 \coloneqq & \mathbf{CohGpd}(\widehat{\mathcal{PA}sm}) \arrow[r, hook] & \mathbf{Gpd}(\widehat{\mathcal{PA}sm})
\end{tikzcd}
$$

Compact groupoids

Say that a compact discrete groupoid is a discrete groupoid $\underline{K}$ on a compact presheaf K.

A groupoid $\mathbb{G}$ is pseudo-compact if there is

$$\underline{K} \to \mathbb{G}$$

essentially surjective on objects from a compact discrete groupoid $\underline{K}$, that is:

$$\begin{tikzcd}
\bullet \arrow[r] \arrow[d] \arrow[rr, bend left, two heads] \arrow[dr, phantom, "\lrcorner", very near start] & G_1 \arrow[r, two heads] \arrow[d] & G_0 \\
K \arrow[r] & G_0 &
\end{tikzcd}$$

NB: A discrete groupoid $\underline{X}$ is pseudo-compact iff X is compact.

Coherent groupoids

A groupoid $\mathbb{G}$ is coherent if

1. $\mathbb{G}$ is pseudo-compact
2. $\Delta_1 : \mathbb{G} \to \mathbb{G} \times \mathbb{G}$ is pseudo-compact:
   for every pseudo-pullback with $\mathbb{C}$ pseudo-compact

$$\begin{tikzcd}
\mathbb{C}' \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner\mathrm{ps}", very near start] & \mathbb{G} \arrow[d, "\Delta"] \\
\mathbb{C} \arrow[r] & \mathbb{G} \times \mathbb{G}
\end{tikzcd}$$

   $\mathbb{C}'$ is also pseudo-compact.
3. $\Delta_2 : \Delta_1 \to \Delta_1 \times_{\mathbb{G}\times\mathbb{G}} \Delta_1$ is pseudo-compact (in the sense above).

NB: A discrete groupoid $\underline{X}$ is coherent iff X is a coherent presheaf.

Recognising coherent groupoids

Lemma

A groupoid $\mathbb{G}$ is coherent if and only if
there are a groupoid $\mathbb{K} = (K_1 \rightrightarrows K_0)$ and a (weak) equivalence $\mathbb{K} \to \mathbb{G}$ such that:

1. K_0 is a compact object: there is P s.t. $yP \twoheadrightarrow K_0$
2. $K_1 \to K_0 \times K_0$ is compact:

   for every K compact and
$$\begin{tikzcd} K' \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & K_1 \arrow[d] \\ K \arrow[r] & K_0 \times K_0 \end{tikzcd}$$
   K' is compact.
3. $K_1 \to K_1 \times_{K_0 \times K_0} K_1$ is compact.

The Quillen model structure on groupoids

We use the strong stacks model structure:

Theorem (Joyal & Tierney)

There is a model structure on $\mathbf{Gpd}(\widehat{\mathcal{PAsm}}) = [\mathcal{PAsm}, \mathbf{Gpd}]$ where:

- The cofibrations are the functors that are objectwise injective on objects.
- The weak equivalences are the functors that are objectwise equivalences of groupoids (= functors that are fully faithful and essentially surjective on objects.)

Joyal, Tierney. Strong stacks and classifying spaces. 1991

This holds for $\mathbf{Gpd}(\mathcal{E})$ with $\mathcal{E}$ a Grothendieck topos.
For $\mathcal{E} = \mathcal{S}et$, it coincides with the folk model structure on $\mathbf{Gpd}$, but it has less fibrations otherwise (since equivalences $\neq$ strong equivalences in a general $\mathcal{E}$).

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This model structure agrees with the restriction to 1-types of the injective model structure on simplicial presheaves used by Shulman to construct univalent universes in $(\infty, 1)$-toposes.

Shulman. All $(\infty, 1)$-toposes have strict univalent universes. 2019

The Quillen model structure on coherent groupoids

Proposition

The two weak factorisation systems from the strong stacks model structure on $\mathbf{Gpd}(\widehat{\mathcal{PAsm}})$ restrict to $\mathbf{CohGpd}(\widehat{\mathcal{PAsm}})$.

Follows from:

Lemma

If $\mathbb{F} \to \mathbb{G}$ is an equivalence of groupoids and $\mathbb{F}$ is coherent, then so is $\mathbb{G}$.

Lemma (Joyal–Tierney)

A trivial fibration is a strong equivalence of groupoids.

The main result

Theorem

Let $\mathbb{G}$ be a coherent groupoid which is also a fibrant 0-type.
Then $\pi_0\mathbb{G}$ is coherent (in $\widehat{\mathcal{PAsm}}$) and $\mathbb{G} \to \underline{\pi_0\mathbb{G}}$ is a strong equivalence.

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Recall that $\mathbb{G}$ is coherent iff there is a weak equivalence $\mathbb{C} \to \mathbb{G}$
with C_0, $C_1 \to C_0 \times C_0$, and $C_1 \to C_1 \times_{C_0 \times C_0} C_1$ compact.

If $\mathbb{G}$ is also a fibrant 0-type, then $G_1 \to G_0 \times G_0$ is monic.

$$\begin{tikzcd}
 & & & G_1 \arrow[rr, two heads] \arrow[dd, tail, red] & & \pi_0\mathbb{G} \arrow[dd, tail] \\
yQ \arrow[r, two heads] \arrow[dr] & K \arrow[r] \arrow[d, tail] \arrow[dr, phantom, very near start, "\lrcorner"] & C_1 \arrow[ur] \arrow[rr] \arrow[d, tail] & & \pi_0\mathbb{C} \arrow[ur, "\sim"] \arrow[d, tail] & \\
 & & & G_0 \times G_0 \arrow[rr, two heads] & & \pi_0\mathbb{G} \times \pi_0\mathbb{G} \\
 & yP \times yP \arrow[r, two heads] & C_0 \times C_0 \arrow[ur] \arrow[rr, two heads] & & \pi_0\mathbb{C} \times \pi_0\mathbb{C} \arrow[ur, "\sim"] &
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 & yP \times yP \arrow[dr, phantom] & & G_0 \times G_0 \arrow[rr, two heads] & & \pi_0\mathbb{G} \times \pi_0\mathbb{G} \\
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Corollary

The full subcategory of $\mathbf{CohGpd}(\widehat{\mathcal{PAsm}})$ on the fibrant 0-types is equivalent to the effective topos:

$$\mathbf{CohGpd}(\widehat{\mathcal{PAsm}})_{0,f} \simeq \mathcal{Coh}(\widehat{\mathcal{PAsm}}) = \mathcal{Eff}$$

$\mathbf{Eff}_2 := \mathbf{CohGpd}(\widehat{\mathcal{PAsm}})$ is the restriction to 1-types of the effective ∞-topos constructed by M. Anel, S. Awodey, R. Barton.

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Thank you!