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Efficient proof systems in provability logics

In this talk I will focus on fragments of provability logics that are well-behaved. An important character in this talk will be the logic **GLP**: the polymodal provability logic introduced by Japaridze [1] where for each natural number n , there is a modality $[n]$ whose intended meaning is a provability predicate. The idea is that the larger the n , the stronger the provability notion as reflected by one of the axioms: $[n]\varphi \rightarrow [m]\varphi$ whenever $n \leq m$.

The logic **GLP** is very similar to the foundational provability logic **GL** and as a matter of fact, for each modality $[n]$ all the axioms of **GL** in this modality are part of **GLP**. There are just two axiom schemes that involve modalities of different sorts. The first is the above mentioned axiom scheme $[n]\varphi \rightarrow [m]\varphi$ whenever $n \leq m$ which expresses some sort of monotonicity: larger numbers correspond to stronger provability notions. The other axiom scheme is $\langle n \rangle \varphi \rightarrow [m] \langle n \rangle \varphi$ which is sometimes called *negative introspection* and can be seen as expressing that larger numbers correspond to strictly stronger provability predicates (recall that over **GL** $\Box \Diamond \varphi$ is equivalent to $\Box \perp$).

The logic **GLP** is decidable and PSPACE complete however, although complete with Topological semantics, it is Kripke incomplete which makes practical work with **GLP** cumbersome. This will be the starting point of this talk. We will see how various ‘fragments’ of **GLP** are better behaved. One of those fragments is the strictly positive fragment with a poly-time decision procedure. Among others we will present a tree-rewriting calculus for this and similar fragments. It seems that even the tree-rewriting calculus is not yet the final station since some form of normalisation in this calculus involve double exponential bounds. The part on tree rewriting is joint work with David Fernández Duque and Sofia Santiago Fernández [2] and I shall be freely mentioning other (joint) papers as well.

[1] G. Japaridze. The polymodal provability logic. In *Intensional logics and logical structure of theories: material from the Fourth Soviet-Finnish Symposium on Logic*. Metsniereba, Telavi, 1988. In Russian.

[2] S. Santiago Fernández, D. Fernández-Duque, and J. J. Joosten. Tree rewriting calculi for strictly positive logics. *arXiv:2504.18240 [cs.LO]*, 2025.